which the expansions are used are varied. In view of the fact that the transfer times, all for the same physical problem, range from 1.33 to 5.83 dimensionless time units (2π such units equals one year), some of the solutions must be quite inaccurate.

This low-thrust orbit transfer problem, from the Earth's heliocentric orbit to the orbit of Mars, idealized as circular and coplanar, with specific values of dimensionless thrust and mass flow rate, has been solved many times previously, using a variety of numerical optimization algorithms.[4-15] Indeed, this problem has served as a standard problem for making comparative assessments of optimization algorithms. The minimum transfer time was shown in early investigations to be 3.32 dimensionless time units (193 days). This minimum transfer time has been corroborated many times subsequently. Only two of the eight transfer times tabulated in Ref. 1 are within 10% of this value. Thus, the optimization algorithm in Ref. 1 does not appear to be performing very well.

Sheela and Ramamoorthy comment that their longest minimum transfer time (338 days), which corresponds to the smallest computed error in satisfying the state differential equations, is quite close to the actual flight time of the Viking-2 spacecraft, from Earth to Mars. They suggest that this close agreement confirms the validity of their 338-day minimum-time solution. The Viking mission involved a ballistic heliocentric transfer, however, rather than a continuous low-thrust transfer. There is no reason to expect the minimum transfer time for a continuously thrusting spacecraft traveling between idealized orbits to match the transfer time for an actual ballistic mission, in which payload delivered into orbit about Mars was maximized, subject to various constraints. For a continuous-thrust transfer, the transfer time is strongly related to the vehicle thrust acceleration. Indeed, the minimum continuous-thrust transfer time can be made arbitrarily short if the thrust acceleration is made sufficiently large and arbitrarily long if the thrust acceleration is made sufficiently small. Thus, any close agreement between a continuous-thrust transfer time obtained in Ref. 1 and the actual Viking-2 transfer time is purely coincidental.

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Reply by Authors to L.J. Wood

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AT the outset we would like to thank Dr. Wood for his comments on our paper.

As can be seen in Table 1 of Ref. 1, the variations in the transfer time t_f for various values of N are less pronounced for the case of $M=4$ than for those of $M=3$. This clearly shows that if the number of functions used for the approximation is increased, the range of variation of t_f will be less. In fact, this number, say M^*, for which the range of variation of t_f for different values of N does not differ should be obtained by the null hypothesis or some other statistical test, and this is not done in the paper.

Regarding the next point raised by Dr. Wood, it is true that the minimum transfer time for this idealized modeling of the orbit transfer problem has been around 3.32 dimensionless units. As for the results tabulated in Ref. 1, the standard deviation of the transfer times taken over different values of N with 3.32 as a mean for $M=3$ and $M=4$ are 1.611 and 0.587, respectively. This clearly shows the convergence trend and that if M is increased further to M^*, defined above, the standard deviation would be still smaller. Thus, it is not true that the algorithm is not performing well.

Finally, we agree with Dr. Wood that the close agreement of our solution for $M=3$ and $N=4$ of Table 1 with that of the actual transfer time for Viking-2 is purely coincidental. In fact, we did not know at the time of our work that the cost criterion for optimization is the payload instead of block time.

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